

Computer Vision: Assignment 1

Due on Thursday, September 1, 2016

Adapted from Computer Vision by Magnus Oskarsson, Lund University.

Instructor: Andrés Marrugo

Laboratorio de Óptica y Procesamiento de Imágenes
Universidad Tecnológica de Bolívar

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1 Instructions

In this assignment you will study the basics of projective geometry. You will study the representations of points lines and planes, as well as transformations. The data for the assignments is available under <http://opilab.unitecnologica.edu.co/computer-vision/assignment1data.zip>

The assignment is due on 2016-09-01 at 11:59 pm. Make sure you answer all questions and provide complete solutions to the exercises. You may hand in hand-written solutions in PDF format. After each exercise there is a gray box with instructions on what should be included in the report. In addition, all the code should be submitted as m-files. Make sure that your matlab scripts are well commented and can be executed directly (that is, without loading any data, setting parameters etc. Such things should be done in the script).

You will have time to work with the assignments during the class sessions. The report should be written individually, however you are encouraged to work together. Keep in mind that everyone is responsible for their own report and should be able to explain all the solutions.

2 Points in Homogeneous Coordinates

Exercise 1. What is the 2D Cartesian coordinates of the points with homogeneous coordinates

$$\mathbf{x}_1 = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}, \quad \text{and } \mathbf{x}_3 = \begin{pmatrix} 4\lambda \\ -2\lambda \\ 2\lambda \end{pmatrix}, \quad \lambda \neq 0? \quad (1)$$

What is the interpretation of the point with homogeneous coordinates

$$\mathbf{x}_4 = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} ? \quad (2)$$

For the report: Answers are enough.

Computer Exercise 1. Write a matlab function `pflat` that divides the homogeneous coordinates with their last entry for points of any dimensionality. (You may assume that none of the points have last homogeneous coordinate zero.) Apply the function to the points in `x2D` and `x3D` in the file `compEx1.mat`, and plot the result.

```
% Useful matlab commands:
x(end,:) % Extracts the last row of x
repmat(a,[m n]) % Creates a block matrix with mn copies of a.
a./b % Elementwise division.
% Divides the elements of a by the corresponding element of b.
plot(a(1,:),a(2,:),'.') % Plots a point at (a(1,i),a(2,i)) for each i.
plot3(a(1,:),a(2,:),a(3,:),'.') % Same as above but 3D.
axis equal % Makes sure that all axes have the same scale.
```

For the report: Submit the m-file, and the plots.

Exercise 2. Compute the homogeneous coordinates of the intersection (in \mathbb{P}^2) of the lines

$$l_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{and } l_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}. \quad (3)$$

What is the corresponding point in \mathbb{R}^2 ?

Compute the intersection (in \mathbb{P}^2) of the lines

$$l_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \text{and } l_4 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}. \quad (4)$$

what is the geometric interpretation in \mathbb{R}^2 ?

Compute the line that goes through the points with Cartesian coordinates

$$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ and } x_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}. \quad (5)$$

Hint: Re-use the calculations from the line intersections above.

For the report: Submit a complete solution.

Exercise 3. The nullspace of a $m \times n$ matrix A is the set

$$\mathcal{N}(A) = \{x \in \mathbb{R}^n; Ax = 0\} \quad , \quad (6)$$

that is, all the x for which the multiplication Ax gives the zero vector. Explain why the intersection point (in homogeneous coordinates) of l_1 and l_2 (from Exercise 2) is in the null space of the matrix

$$M = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (7)$$

The null space of M is a linear space in \mathbb{R}^3 (see your linear algebra book or wikipedia), what is its dimension? Are there any other points in the null space besides the intersection point?

For the report: Answers all the questions.

Computer Exercise 2. Load and plot the image in `compEx2.jpg` (see figure 1). In the file `compEx2.mat` there



Figure 1: `compEx2.jpg`

are three pairs of image points. Plot the image points in the same figure as the image.

For each pair of points compute the line going through the points. Use the function `rital` to plot the lines in the same image. Do these lines appear to be parallel (in 3D)?

Compute the point of intersection between the second and third line (the lines obtained from the pairs `p2` and `p3`). Plot this point in the same image.

The distance between a 2D-point $x = (x_1, x_2)$ in Cartesian coordinates and a line $l = (a, b, c)$ can be computed using the distance formula

$$d = \frac{|ax_1 + bx_2 + c|}{\sqrt{a^2 + b^2}} \quad (8)$$

see your linear algebra book. Compute the distance between the first line and the the intersection point. Is it close to zero? Why/why not?

```

% Useful matlab commands:

imread('compEx2.JPG') % Loads the image compEx2.JPG

imagesc(im) % Displays the image

colormap gray % changes the colormap of the current image to gray scale

hold on % Prevents the plot command from clearing the figure before plotting hold off %Makes ...
        the plot command clear the figure before plotting

null(A) % computes the nullspace of A

```

Submit the m-file, the plot, and the computed distance between the intersection point and the first line.

3 Projective Transformations

Exercise 4. Let H be the projective transformation

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}. \quad (9)$$

Compute the transformations $\mathbf{y}_1 \sim H\mathbf{x}_1$ and $\mathbf{y}_2 \sim H\mathbf{x}_2$ if

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \text{ and } \mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}. \quad (10)$$

Compute the lines l_1, l_2 containing $\mathbf{x}_1, \mathbf{x}_2$, and $\mathbf{y}_1, \mathbf{y}_2$, respectively.

Compute $(H^{-1})^T l_1$ and compare to l_2 .

For the report: Submit the answers.

Computer Exercise 3. The file `compEx3.mat` contains the start and end points of a set of lines. Plotting the lines gives the grid in figure 2.

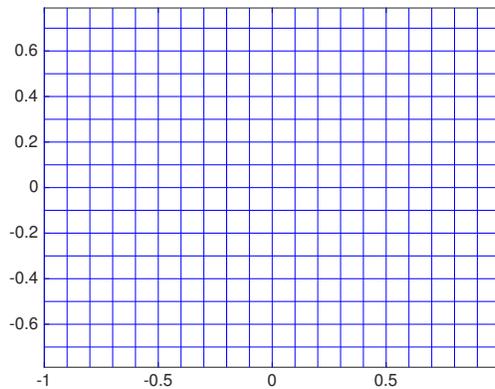


Figure 2: Lines between start and end points.

For each of the projective mappings given by the matrices

$$H_1 = \begin{pmatrix} \sqrt{3} & -1 & 1 \\ 1 & \sqrt{3} & 1 \\ 0 & 0 & 2 \end{pmatrix}, \text{ and } H_2 = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

$$H_3 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ and } H_4 = \begin{pmatrix} \sqrt{3} & -1 & 1 \\ 1 & \sqrt{3} & 1 \\ 1/4 & 1/2 & 2 \end{pmatrix}, \quad (12)$$

compute the transformations of the given start and endpoints and plot the lines between them. (Note that you do not need to loop over the points. One matrix multiplication for the start and end points is enough. To compute cartesian coordinates you can use your `pflat` function. Don't forget to use the `axis equal` command, otherwise the figures might look distorted.)

Which of the transformations preserve lengths between points? Which preserve angles between lines? Which maps parallel lines to parallel lines? Classify the transformations into euclidean, similarity, affine and projective transformations.

```
% Useful matlab commands:
plot([startpoints(1,:); endpoints(1,:)], ...
     [startpoints(2,:); endpoints(2,:)], 'b-');
% Plots a blue line between each startpoint and endpoint
```